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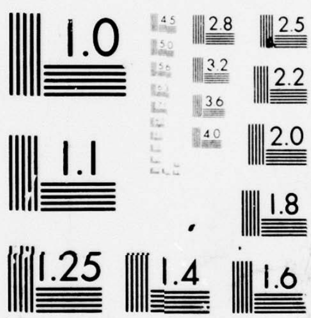

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Fifteen papers were prepared for publication or published during the ensuing period of research. Topics included identification and control of systems with delays, qualitative theory of functional differential and difference equations, nonlinear oscillations, control of dynamical systems, and stability of infinite-dimensional systems.

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CONTROL OF DYNAMICAL SYSTEMS

for the period

Sept. 1, 1978 - Aug. 30, 1979

Lefschetz Center for Dynamical Systems
Division of Applied Mathematics
Brown University
Providence, R. I. 02912

Principal Investigators:
H. T. Banks
J. K. Hale
E. F. Infante
J. P. LaSalle

Report prepared by:
H. T. Banks

October 30, 1979

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I. Identification and control of systems with delays (H.T. Banks)

Banks is continuing his efforts on approximation of delay systems with applications to optimal control and parameter identification. In a manuscript [1] that will appear soon, Banks and Kappel developed the foundations for general spline approximation schemes for linear functional differential equations. Banks, in [2] and in joint efforts with Burns and Cliff [3], has tested these approximations (with simple numerical examples) when applied to control and parameter identification problems with linear delay systems. Banks has also conducted preliminary numerical tests with the spline approximation ideas for nonlinear control systems.

For the parameter identification problems, some rigorous theoretical results for some of the approximation schemes under investigation have been obtained. Especially interesting are results for problems where the delays themselves must be identified. A manuscript on these initial theoretical results is under preparation and we are currently developing software packages to test numerical performance of the related algorithms.

Banks and a graduate student, Rosen, have made good progress on development of fully discretized methods (as opposed to those of [1], [2], [3] which involve approximation of the infinite dimensional delay systems by a finite number of ordinary differential equations). Theoretical results (and the corresponding numerical tests) have been obtained for the basic approximation schemes.

These indicate that feasible methods can be developed but application of the ideas to control and identification problems, along with development of efficient software packages, are efforts still to be pursued in the near future.

II. Qualitative theory of functional differential and difference equations (J. K. Hale)

Hale is continuing his study of the effects of delays on difference and differential-difference equations. For the autonomous linear difference equation

$$x(t) - \sum_{j=1}^{N} A_k x(t-\gamma_k \cdot r) = 0$$

$$\gamma_k \cdot r = \sum_{j=1}^{M} \gamma_{k_j} r_j, \quad \gamma_{k_j} \geq 0, \quad \text{integers},$$

Avellar and Hale [4] have given a rather complete characterization of the behavior of the set

$$\overline{Z}(r,A) = \{\mathrm{Re}\lambda : \det[I - \sum_{k=1}^{N} A_k e^{-\lambda\gamma_k \cdot r}] = 0\}$$

as a function of the delays r and the coefficients A. Also, effective computational criterion have been given. Specific emphasis is given the stable case where $\overline{Z}(r,A) \subseteq (-\infty,0)$. The results have already been applied by Hale and de Oliveira [5] to the Hopf bifurcation in difference integral equations of the form

$$x(t) - g(x(t-r_1),\ x(t-r_2),\ \int_{-r_3}^{0} A(\theta)h(x(t+\theta))d\theta) = 0$$

where g,h are nonlinear functions. José de Oliveira [6] has also applied these results to the same problem for neutral equations.

Hale, Infante and Tsen have determined the region in the coefficient space for which all solutions of the delay equation

$$\frac{dx}{dt} = B_0 x(t) + \sum_{k=1}^{N} B_k x(t-\gamma_k \cdot r)$$

approach zero as $t \to \infty$, independent of the delays. A preliminary manuscript [7] has been prepared. (See additional comments below.)

III. Nonlinear Oscillations (J. K. Hale)

Hale is continuing to work on nonlinear oscillations in evalutionary equations. Even though the original evolutionary equation may be infinite dimensional (a system of nonlinear parabolic equations, functional differential equations), the basic work in the literature is still devoted to low finite dimensional problems (dimension one, two and ocassionally three) which are obtained by means of the center manifold theorem. Some basic questions remain unresolved even in these lower dimensional spaces.

During the last year, Hale and his colleagues have had success on two main problems. The first one has to do with determining the dynamic behavior of a periodic system from the bifurcation equation obtained via an application of the method of Liapunov-Schmidt. We describe the results in $\mathbb{R}^n$ although they are applicable more generally. Consider the equation

$$(1) \qquad \dot{x} = Ax + f(t,x,\lambda) \quad , \quad A = \begin{pmatrix} 0 & 0 \\ 0 & B \end{pmatrix}$$

where λ is a vector parameter, A is $(n+1) \times (n+1)$, B is $n \times n$ with eigenvalues with negative real parts, $f(t+2\pi,x,\lambda) = f(t,x,\lambda)$, $f(t,0,0) = 0$, $\partial f(t,0,0)/\partial x = 0$. The method of Liapunov-Schmidt is easily applied to obtain a scalar bifurcation function $G(a)$ whose zeros are in a one-to-one correspondence with the 2π-periodic solutions of (1) near $x = 0$, $\lambda = 0$. Hale and de Oliveira [8] prove that the stability properties of the 2π-periodic solutions of (1) are completely determined by the stability properties of the corresponding zero of G as a solution of the scalar equation

$$\dot{a} = G(a).$$

This result gives new results on stability even when f is independent of t. When applied to the problem of Hopf bifurcation, it completely eliminates the consideration of averaging which is more difficult to apply. Also, it gives results on multiple parameter bifurcation which cannot be obtained by averaging.

Chow, Hale and Mallet-Paret are in the process of completing part of their work on the existence of homoclinic points for a second order equation

$$\ddot{x} + g(x) = \lambda_1 \dot{x}(t) + \lambda_2 f(t)$$

where $f(t+1) = f(t)$. They have given generic conditions for the existence of homoclinic points and have discussed subharmonic bifurcations near homoclinic points.

IV. Control of dynamical systems (J. P. LaSalle)

1. Continuous-time vs. discrete-time models.

There have been over a long period of time discussions of the suitability of continuous-time vs. discrete-time systems as mathematical models for various types of applications. A most recent example is an unpublished paper by Kenneth L. Cooke (Pomona College and Brown University), "On the Construction and Evaluation of Mathematical Models". Cooke says: "If discrete time steps are selected, we encounter difference equations, if continuous-time is chosen, we have differential equations. Will the results be the same? In an abstract mathematical sense, this is nearly true, but in practice caution is required." Cooke's statement "nearly true" is based on approximating the derivative by a difference quotient. This is often the way in which discrete models are obtained and is a poor way indeed, of selecting a suitable discrete model and of judging the suitability of a discrete model.

The fact of the matter is that in a certain sense there is always a discrete-time model that is exactly equivalent to the continuous-time model. Every continuous-time dynamical system (or semi-dynamical system or process), if sampled (an infinite sequence of sample times), can be represented exactly by a system of difference equations. If the continuous-time system is autonomous and the sampled times are evenly spaced, then the difference equations will also be autonomous.

The proof of the above statement is so simple that the conclusion can be said to be merely observation. This observation does not answer the basic question of the relativ e merits of the two types of models. It does, however, raise considerably the level of discussion, shows that some arguments used against discrete models are invalid, and makes clear that the derivation of suitable discrete-time models should be direct (not necessarily involving continuous-time reasoning). LaSalle is studying these questions.

2. Discrete control

LaSalle has obtained some new results in determining feedback controls that perform an operation in minimum time and do this with the minimum amount of energy.

3. Multiflows

The determinism of classical physics and classical science has always strongly motivated the geometric theory of differential equations and dynamical systems to the extent that it is still not realized how much of the theory is independent of the assumption of the uniqueness of solutions. One of the most important and natural examples of a nondeterministic system is a system under control. A control parameter may determine a variety of solutions.

LaSalle and Artstein are completing a study whose purpose is to develop a language for describing and identifying the common structural features of systems without uniqueness. Their abstract concept (called a multiflow) is described in terms of basic properties of "solution funnels" rather than the attainable sets of Barbashin, Roxin, Szego and Treccani, and Kloeden, among others. It differs also from the approach taken by Sell [1973] and is closer to the ideas of Yorke [1969].

V. Stability of infinite-dimensional systems (E. F. Infante)

Infante and collaborators have continued to pursue research centered on the stability properties of infinite-dimensional systems, with particular emphasis on certain problems arising in certain applications.

In cooperation with J.K. Hale and a student, F.P. Tsen, Infante [7] has completed an investigation on the effects of changes in the delays on the stability properties of linear neutral difference-differential equations of the form

$$\frac{d}{dt}\left[x(t) + \sum_{k=1}^{n} B_k x(t-\tau_k)\right] = A_0 x(t) + \sum_{k=1}^{n} A_k x(t-\tau_k).$$

The effect of changes of the values of the $\{\tau_k\}_{k=1}^n$ on the asymptotic behavior of the solutions of such equations is most important; indeed, it is not necessarily the case that the asymptotic behavior varies continuously with changes in the $\{\tau_k\}_{k=1}^n$. Moreover, the computation of the effect of such changes is most awkward and complicated since it involves the computation of zeros of exponential polynomials. The purpose of the investigation reported in [7] was to characterize in an appropriate and convenient manner those matrices $\{B_k\}_{k=1}^n$, A_0 and $\{A_k\}_{k=1}^n$ such that the solutions of the above equation do not change their asymptotic behavior irrespective of the values of the delays $\{\tau_k\}_{k=1}^n$. Such a characterization seems important in the design and control of systems with delays; if the model of the system falls within the characterized class, it will display considerable robustness; if it does not, considerable care must be exercised in analyzing modeling errors.

Infante, in cooperation with a student, L. Carvalho, has completed an investigation [9] of the stability properties of pure linear difference equations of the form

$$x_t(0) = \sum_{k=1}^{n} A_k x_t(-\tau_k),$$

where the $\{\tau_k\}_{k=1}^{n}$ are not necessarily rationally related, and therefore the solution of such equations cannot be viewed in a finite-dimensional space. By viewing the solutions of such problems in a convenient Hilbert space, sufficient conditions for the existence of a particular type of Liapunov functionals, which prove stability and asymptotic stability for such equations irrespective of the value of the delays, were found. Moreover, a characterization of the $\{A_k\}_{k=1}^{n}$ for which such functionals exist is also presented. Such results seem of importance not only for stability considerations but also for optimization purposes, since most often the functionals one is required to minimize are precisely of the form of the Liapunov functionals considered.

In cooperation with J. A. Walker and L. Carvalho, Infante has completed an investigation [10] on the existence of simple Liapunov functionals for linear retarded difference differential equations of the form

$$\dot{y}(t) = Ay(t) + \sum_{k=1}^{m} B_k y(t-\tau_k).$$

The question refers to the existence of functionals of the two forms

$$V(x) = y^*Ry + \sum_{k=1}^{m} \int_{-\tau_k}^{0} \dot{v}^*(\theta)Q_k v(\theta)d\theta,$$

and

$$W(x) = \sup_{\max \tau_k \leq \theta \leq 0} v^*(\theta)Rv(\theta).$$

It is noted that the existence of a Liapunov function of either of the foregoing forms is independent of the delays $\{\tau_k\}_{k=1}^{m}$; hence so will be any stability conclusions drawn from the use of such functions. This remark suggests that the class of functional equations of the above form for which such functions exist is narrow. In [10] a characterization of this class is given; moreover, it is shown by counterexample that there do exist equations of this form that yield asymptotic stability for all delays $\{\tau_k\}_{k=1}^{m}$ and yet there exist no Liapunov function of the above forms that can be used to establish this fact. Finally, the Liapunov functions of the above type are used to provide estimates of exponential decay.

REFERENCES

[1] H.T. Banks and F. Kappel, Spline approximations for functional differential equations, to appear in J. Diff. Eqns., Nov. 1979.

[2] H.T. Banks, Approximation of delay systems with applications to control and identification, Functional Differential Equations and Approximation of Fixed Points (H.-O. Peitgen and H.-O. Walther, eds.), Lecture Notes in Math., Vol. 730, Springer-Verlag, Berlin, 1979, pp. 65-76.

[3] H.T. Banks, J.A. Burns, and E.M. Cliff, Spline-based approximation methods for control and identification of hereditary systems, to appear in, Proc. Intl. Symposium on Systems Optimization and Analysis, IRIA, France, Dec. 11-13, 1978.

[4] C.E. Avellar and J.K. Hale, On the zeros of exponential polynomials, to appear in JMAA.

[5] J.C.F. de Oliveira and J.K. Hale, Hopf bifurcation for functional equations, to appear in JMAA.

[6] J.C.F. de Oliveira, Hopf bifurcation for functional differential equations, to appear in JNA:TMA.

[7] J.K. Hale, E.F. Infante and F.P. Tsen, Stability in delay equations for all delays, to be submitted.

[8] J.C.F. de Oliveira and J.K. Hale, Dynamic behavior from bifurcation equations, special issue, Tohoku Mathematical Journal (Japan) dedicated to T. Yoshizawa, to appear.

[9] L. Carvalho and E.F. Infante, On Liapunov functionals for linear difference equations, to be submitted.

[10] L. Carvalho, E.F. Infante and J.A. Walter, On the existence of simple Liapunov functions for linear retarded difference differential equations, to appear, Tohoku Jour. of Math.

PUBLICATIONS SUPPORTED BY THE AIR FORCE

SEPTEMBER 1978 - AUGUST 31, 1979

[1] H.T. Banks (with J.A. Burns and E.M. Cliff)

Spline-based approximation methods for control and identification of hereditary systems, to appear, Proc. Intl. Symp. on Systems Optimization & Anal., IRIA, France, December 11-13, 1978.

[2] H.T. Banks (with Franz Kappel)

Spline approximations for functional differential equations, Journal of Differential Equations (to appear).

[3] H.T. Banks

Approximation of delay systems with applications to control and identification, in Functional Differential Equations and Approximation of Fixed Points (H.-O. Peitgen and H.-O. Walther, eds.), Lecture Notes in Mathematics, Vol. 730, Springer-Verlag, Berlin, 1979, pp. 65-76.

[4] J.K. Hale (with J.C.F. deOliveira)

Hopf bifurcation for functional equations, Journal of Mathematical Analysis and Applications (to appear).

[5] J.K. Hale (with J.C.F. deOliveira)

Dynamic behavior from bifurcation equations. Paper dedicated to Professor Taro Yoshizawa's 60th birthday. To appear in a special issue, Tohoku Mathematical Journal.

[6] J.K. Hale

Some recent results on dissipative processes (to be submitted).

[7] J.K. Hale (with C.E. Avellar)

On the zeros of exponential polynomials, Journal of Mathematical Analysis and Applications (to appear).

[8] J.K. Hale

Topics in local bifurcation theory. Annals of the New York Academy of Sciences, vol. 316, pp. 605-607 in Bifurcation Theory and Applications in Scientific Disciplines (Okan Gurel and Otto E. Rossler, Eds.), 1979.

[9] J.K. Hale

Bifurcation from simple eigenvalues for several parameter families. Nonlinear Analysis Theory, Methods and Applications, vol. 2, no. 4, pp. 491-497, 1978.

[10] J.K. Hale (with Junji Kato)

Phase space for retarded equations with infinite delay. Funkcialaj Ekvacioj, vol. 21, no. 1, December 1978, pp. 11-41.

[11] E.F. Infante (with L.A.V. Carvalho and J.A. Walker)

On the existence of simple Liapunov functions for linear retarded difference differential equations. Paper dedicated to Professor Taro Yoshizawa's 60th birthday. To appear in a special issue, Tohoku Mathematical Journal.

[12] E.F. Infante (with W.B. Castelan)

A Liapunov functional for a matrix difference-differential equation. Journal of Differential Equations, vol. 29, no. 3, September 1978, pp. 439-451.

[13] Paul Massatt

Stability and fixed points of point dissipative systems, Journal of Differential Equations (to appear).

[14] Paul Massatt

Some properties of condensing maps, Annali di Matematica Pura e Applicada (to appear).

[15] Lawrence Turyn

Sturm-Liouville problems with several parameters, submitted to the Journal of Differential Equations.

Spline-Based Approximation Methods
for Control and Identification of Hereditary Systems

H. T. Banks, J. A. Burns and E. M. Cliff

Abstract

Convergence results for schemes based on first-order spline approximations are presented for optimal control and parameter identification problems involving linear delay-differential equations. Examples with numerical results which demonstrate the attractiveness of the proposed methods are given.

SPLINE APPROXIMATIONS

FOR

FUNCTIONAL DIFFERENTIAL EQUATIONS

H. T. Banks and F. Kappel

Abstract: We develop an approximation framework for linear hereditary systems which includes as special cases approximation schemes employing splines of arbitrary order. Numerical results for first and third order spline based methods are presented and compared with results obtained using a previously developed scheme based on averaging ideas.

APPROXIMATION OF DELAY SYSTEMS WITH APPLICATIONS TO CONTROL AND IDENTIFICATION

H. T. Banks

Abstract: We discuss approximation ideas for functional differential equations and how these ideas can be employed in optimal control and parameter estimation problems. Two specific schemes are described, one based on integral averages of the function being approximated, the other based on best L_2 spline approximations. An example illustrating numerical behavior of these schemes applied to an optimal control problem is presented.

HOPF BIFURCATION FOR FUNCTIONAL EQUATIONS

J.K. HALE AND J.C.F. deOLIVEIRA

Abstract: The purpose of this paper is to study the existence of a smooth Hopf bifurcation for functional equations. The bifurcation parameters may include the delays. The results will be described for a special case of the equations considered.

DYNAMIC BEHAVIOR FROM BIFURCATION EQUATIONS

Jose C. Fernandes de Oliveira
and
Jack K. Hale

ABSTRACT

Necessary and sufficient conditions for existence of small periodic solutions of some evolution equations can be obtained by the Liapunov-Schmidt Method. In a neighborhood of zero, this gives a function (the bifurcation function) to each zero of which corresponds a periodic solution of the original equations. If this function is scalar, we show that its sign between the zeros gives the complete description of the stability properties of the periodic solutions.

Dedicated to Professor Taro Yoshizawa on his 60th birthday.

SOME RECENT RESULTS ON DISSIPATIVE PROCESSES

JACK K. HALE

ABSTRACT: The literature on dissipative processes is mainly concerned with the following questions:

(1) What conditions imply the existence of a maximal compact invariant set?

(2) What are the stability properties of this set?

(3) What is the relationship between different types of attractivity of this maximal compact invariant set? For example, when is asymptotic stability (stable and point dissipative) equivalent to uniform asymptotic stability (stable and local compact dissipative)? When is the property of this maximal compact invariant set being asymptotically stable equivalent to the property that it attracts bounded sets of X.

(4) When does T have a fixed point?

The purpose of this paper is to present some of the recent results on the above questions. At the same time, we take the opportunity to correct a few mistakes and give a few new implications of the results. The hypotheses for the results will involve certain attractivity and dissipative properties as well as some smoothness properties of T.

ON THE ZEROS OF EXPONENTIAL POLYNOMIALS

Cerino E. Avellar and Jack K. Hale

ABSTRACT: Suppose $r = (r_1, \ldots, r_M)$, $r_j \geq 0$, $\gamma_{kj} \geq 0$ integers, $k = 1,2,\ldots,N$, $j = 1,2,\ldots,M$, $\gamma_k \cdot r = \sum_j \gamma_{kj} r_j$. The purpose of this paper is to study the behavior of the zeros of the function

$$h(\lambda, a, r) = 1 + \sum_{j=1}^{N} a_j e^{-\lambda \gamma_j \cdot r}$$

where each a_j is a nonzero real number. More specifically, if $\overline{Z}(a,r) = \text{closure}\{\text{Re}\,\lambda : h(\lambda,a,r) = 0\}$, we study the dependence of $\overline{Z}(a,r)$ on a, r. This set is continuous in a but generally not in r. However, it is continuous in r if the components of r are rationally independent. Specific criterion to determine when $0 \notin \overline{Z}(a,r)$ are given. Several examples illustrate the complicated nature of $\overline{Z}(a,r)$.

The results have immediate implication to the theory of stability for difference equations

$$x(t) - \sum_{k=1}^{M} A_k x(t - r_k) = 0$$

where x is an n-vector, since the characteristic equation has the form given by $h(\lambda, a, r)$. The results give information about the preservation of stability with respect to variations in the delays.

The results also are fundamental for a discussion of the dependence of solutions of neutral differential difference equations on the delays. These implications will appear elsewhere.

TOPICS IN LOCAL BIFURCATION THEORY

by

Jack K. Hale

Abstract

Suppose Λ, X, Z are Banach spaces, $M: \Lambda \times X \to Z$ is a mapping continuous together with derivatives up through some order r. A bifurcation surface for the equation (1) $M(\lambda, x) = 0$ is a surface in parameter space Λ for which the number of solutions x of (1) changes as λ crosses this surface. Under certain generic hypotheses on M, the author and his colleagues have shown that one can systematically determine the bifurcation surfaces by elementary scaling techniques and the implicit function theorem. This talk gives a summary of these results for the case of bifurcation near an isolated solution or families of solutions of the equation $M(\lambda_0, x) = 0$. The results have applications to the buckling theory of plates and shells under the effect of external forces, imperfections, curvature and variations in shape. The results on bifurcation near families has applications in nonlinear oscillations and the theory of homoclinic orbits.

PHASE SPACE FOR RETARDED EQUATIONS WITH INFINITE DELAY

Jack K. Hale and Junji Kato

Abstract: It is the purpose of this paper to examine initial data from a general Banach space. We develop a theory of existence, uniqueness, continuous dependence, and continuation by requiring that the space B only satisfies some general qualitative properties. Also, we impose conditions of B which will at least indicate the feasibility of a qualitative theory as general as the one presently available for retarded equations with finite delay in the space of continuous functions. In particular, this will imply that bounded orbits should be precompact and that the essential spectrum of the solution operator for a linear autonomous equation should be inside the unit circle for $t > 0$. Also, we impose conditions which imply the definitions of asymptotic stability in R^n and B are equivalent and that the ω-limit set of a precompact orbit for an autonomous equation should be compact, connected and invariant.

ABSTRACT

ON THE EXISTENCE OF SIMPLE LIAPUNOV FUNCTIONS FOR LINEAR RETARDED DIFFERENCE DIFFERENTIAL EQUATIONS

by

L.A.V. Carvalho, E.F. Infante, J. A. Walker

Liapunov functions of simple form have been used for the study of stability properties of difference-differential equations. In this paper we provide necessary and sufficient conditions for the existence of such functions.

A LIAPUNOV FUNCTIONAL FOR A MATRIX DIFFERENCE-DIFFERENTIAL EQUATION

E.F. Infante and W.B. Castelan

Abstract: A quadratic positive definite functional that yields necessary and sufficient conditions for the asymptotic stability of the solutions of the matrix difference-differential equation $\dot{x}(t) = Ax(t) + Bx(t-\tau)$ is constructed and its structure is analyzed. This functional, a Liapunov functional, provides the best possible estimate for the rates of growth or decay of the solutions of this equation. The functional obtained, and its method of construction, are natural generalizations of the same problem for ordinary differential equations, and this relationship is emphasized. An example illustrates the applicability of the results obtained.

STABILITY AND FIXED POINTS OF POINT DISSIPATIVE SYSTEMS

Paul Massatt

Abstract: It is known that if $T: X \to X$ is completely continuous or if there exists an $n_0 > 0$ such that T^{n_0} is completely continuous, then T point dissipative implies T is bounded dissipative and has a fixed point (see Billotti and LaSalle [1]). This result is used, for example, in studying retarded functional differential equations.

This result has been extended by Hale and Lopes [8]. They get the result that if T is an α-contraction and compact dissipative then T is bounded dissipative and has a fixed point. This applies, for example, to stable neutral functional differential equations and certain retarded functional differential equations of infinite delay. These results are contained in Hale [5].

The above result requires the stronger assumption of compact dissipative. The principle result of this paper will be to get similar results under the weaker assumption of point dissipative. We will need to add additional hypotheses on the space and the operator T. We will then show how these hypotheses are naturally satisfied for stable neutral functional differential equations and retarded functional differential equations of infinite delay.

The paper will be divided into four sections. The first will contain various definitions. The second will contain an abstract theorem relating point dissipative in one space to bounded dissipative in another, and to the existence of a fixed point. The third section will apply the result to stable neutral functional differential equations. The final section gives applications to retarded functional differential equations with infinite delay.

This paper is a part of my thesis at Brown University. I am especially grateful to Jack K. Hale for his help and supervision in the preparation of this paper.

SOME PROPERTIES OF CONDENSING MAPS

Paul Massatt

Abstract: In 1939, Kuratowskii introduced a measure of non-compactness of bounded sets in a metric space, called the Kuratowskii measure of noncompactness, or α-measure. This along with the associated notion of an α-contraction, has proved useful in several areas of differential equations. The principle results of this paper will be to generalize several of the results of Cooperman to more general measures of noncompactness as well as for certain set mappings. The proofs are more elementary than the ones in Cooperman. However, the basic lemma used by Cooperman which depended so much on properties of α-measures is not generalized. In fact, we give an example showing that it will not generally be valid for arbitrary measures of noncompactness.

STURM-LIOUVILLE PROBLEMS WITH SEVERAL PARAMETERS

LAWRENCE TURYN

ABSTRACT

We consider the regular linear Sturm-Liouville problem (second-order linear ordinary differential equation with boundary conditions at two points $x = 0$ and $x = 1$, those conditions being separated and homogeneous) with several real parameters $\lambda_1,\ldots,\lambda_N$. Solutions to this problem correspond to eigenvalues $\lambda = (\lambda_1,\ldots,\lambda_N)$ lying on surfaces in $\mathbb{R}^N$ determined by the number of zeroes in $(0,1)$ of solutions. We describe properties of these surfaces, including: boundedness, and when unbounded, asymptotic directions. Using these properties some results are given for the system of N Sturm-Liouville problems which share only the parameters λ. Sharp results are given for the system of two problems sharing two parameters.

The eigensurfaces for a single problem are closely related to the cone $K = \{\lambda \in \mathbb{R}^N: \lambda_1 a_1(x) + \ldots + \lambda_N a_N(x) \leq 0$ for all x in $[0,1]\}$, particularly in questions of boundedness. The cone K and related objects are discussed, and a result is given which relates cones with two oscillation conditions known as "Right-Definiteness" and "Left-Definiteness".